A Formal Summary of Variational Induction

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1 Introduction

Variational induction is an inductive method based on a difference-making logic. In the following, the account of variational induction developed in Pietsch (2016; 2022, Chs. 5,6) is summarized from a formal perspective.[1]

2 Basic Definitions

A.2.1 Causal relevance: *In a context B, in which a condition or circumstance A and a phenomenon C occur, A is 'causally relevant' to C, in short A* $\mathcal{R}$ *C | B, iff the following counterfactual holds: if A had not occurred in context B, C would also not have occurred.*

Counterfactual: *'If A were not the case, C would not be the case' is true with respect to an instance in which both A and C occur in a context B, iff (1) at least one instance is realized in the actual world in which neither A nor C occurs in the same context B and (2) B guarantees homogeneity.*

Formally: $A \, \mathcal{R} \, C \mid B \Leftrightarrow \exists \, \{A \wedge C \, ; \neg A \wedge \neg C \mid B\}$ <D1>

Example: Virus Present $\mathcal{R}$ Disease Present | B $\Leftrightarrow \exists$ {(Virus Present) $\wedge$ (Disease Present); (Virus Absent $\wedge$ Disease Absent) | B }

A.2.1.1 Causal relevance of $A \wedge X$

$A \wedge X \, \mathcal{R} \, C \mid B$

$\Leftrightarrow \exists \, \{A \wedge X \wedge C \, ; \neg (A \wedge X) \wedge \neg C \mid B\}$

$\Leftrightarrow \exists \, \{A \wedge X \wedge C; (\neg A \vee \neg X) \wedge \neg C) \mid B \}$

$\Leftrightarrow \exists \, \{A \wedge X \wedge C; \neg A \wedge X \wedge \neg C; A \wedge \neg X \wedge \neg C; \neg A \wedge \neg X \wedge \neg C \mid B \}$ <D1*>

The last step is based on the convention that in case of an $\vee$ - combination all possible instances shall be realized.

[1] Accounts of induction based on a difference making logic have a long history in debates on scientific method (e.g. Bacon 1620, Bk. II; Herschel 1830, P. II Ch. VI; Mill 1843, Bk. III Ch. VIII; Mackie 1967, Appendix; Baumgartner & Graßhoff 2004). This type of induction has been called variational or variative induction (Cohen 1989; cf. Russo 2007, 2009). It has been used in the analysis of big data practices (Pietsch 2021), including large language models (Pietsch 2026).

A.2.1.2 Causal relevance of A ∨ X

$A \vee X\ \mathcal{R}\ C \mid B$

$\Leftrightarrow \exists \{(A \vee X) \wedge C\ ;\ \neg (A \vee X) \wedge \neg C \mid B\}$

$\Leftrightarrow \exists \{(A \vee X) \wedge C;\ (\neg A \wedge \neg X) \wedge \neg C) \mid B \}$

$\Leftrightarrow \exists \{A \wedge X \wedge C;\ \neg A \wedge X \wedge C;\ A \wedge \neg X \wedge C;\ \neg A \wedge \neg X \wedge \neg C \mid B \}$ <D1**>

In the last step the same convention as in the derivation of <D1*> was used.

A.2.2 Causal irrelevance: *In a context B, in which a condition or circumstance A and a phenomenon C occur, A is 'causally irrelevant' to C, in short A* $\mathcal{I}$ *C | B, iff the following counterfactual holds: if A had not occurred, C would still have occurred.*

Counterfactual: *'If A were not the case, C would still be the case' is true with respect to an instance in which both A and C occur in a context B, iff (1) there exists at least one instance in which A does not occur but C still occurs in the same context B and (2) B guarantees homogeneity.*

Formally: $A\ \mathcal{I}\ C \mid B \Leftrightarrow \exists \{A \wedge C\ ;\ \neg A \wedge C \mid B \}$ <D2>

Example: Full Moon Present $\mathcal{I}$ Disease Present | B ⇔ ∃ {(Full Moon Present) ∧ (Disease Present) ; (Full Moon Absent) ∧ (Disease Present) | B }

A.2.3 Causal factor: *A is a 'causal factor' for phenomenon C with respect to background B, iff there exists an X such that A is causally relevant to C with respect to B∧X and irrelevant to ¬C with respect to B∧¬X (i.e. C is always absent in B∧¬X).*

Formally: $A\ \mathcal{R}_{cf}\ C \mid B \Leftrightarrow \exists X \rightarrow A\ \mathcal{R}\ C \mid B \wedge X$ and $A\ \mathcal{I}\ \neg C \mid B \wedge \neg X$ <D3>

Note that for A to be a causal factor X cannot be redundant. In other words, if A is relevant as a causal factor to C with respect to B, it cannot at the same time be causally relevant to C with respect to B.

Example: Ignited Match Present $\mathcal{R}_{cf}$ Fire Present | *B* ⇔ ∃ X (Combustible Material Present) → Ignited Match Present $\mathcal{R}$ Fire Present | B ∧ Combustible Material Present AND Ignited Match Present $\mathcal{I}$ Fire Absent | B ∧ Combustible Material Absent

A.2.4 Alternative cause: *A is an 'alternative cause' to C with respect to background B iff there exists an X such that A is causally relevant to C with respect to a background B∧¬X, but causally irrelevant to C with respect to a background B∧X (i.e. C is always present in B∧X).*

Formally: $A \mathcal{R}_{ac} C \mid B \Leftrightarrow \exists X \rightarrow A \mathcal{R} C \mid B \wedge \neg X$ and $A \mathcal{I} C \mid B \wedge X$ <D4>

Note that for A to be an alternative cause X cannot be redundant. In other words, if A is relevant as an alternative cause to C with respect to B, it cannot at the same time be causally relevant to C with respect to B.

Example: Sprinkler On $\mathcal{R}_{ac}$ Lawn Wet | $B \Leftrightarrow \exists$ X (Rain Present) → Sprinkler On $\mathcal{R}$ Lawn Wet | B ∧ Rain Absent AND Sprinkler On $\mathcal{I}$ Lawn Wet | B ∧ Rain Present

A.2.5 Inus-complex: *A complex of conditions of the form (X∧Y)∨Z shall be called an 'inus-complex' for C with respect to B, iff it is causally relevant to C with respect to B. Note that each condition X, Y, or Z may itself be an inus-complex. By definition, it may be the case that Y=1 and/or Z=0.*

$(X \wedge Y) \vee Z \mathcal{R} C \mid B$

$\Leftrightarrow \exists \{(X \wedge Y) \wedge Z \wedge C; \neg (X \wedge Y) \wedge Z \wedge C; (X \wedge Y) \wedge \neg Z \wedge C; \neg (X \wedge Y) \wedge \neg Z \wedge \neg C \mid B \}$ (based on <D1**>)

$\Leftrightarrow \exists \{X \wedge Y \wedge Z \wedge C; (\neg X \vee \neg Y) \wedge Z \wedge C; X \wedge Y \wedge \neg Z \wedge C; (\neg X \vee \neg Y) \wedge \neg Z \wedge \neg C) \mid B \}$

$\Leftrightarrow \exists \{X \wedge Y \wedge Z \wedge C; \neg X \wedge \neg Y \wedge Z \wedge C; \neg X \wedge Y \wedge Z \wedge C; X \wedge \neg Y \wedge Z \wedge C; X \wedge Y \wedge \neg Z \wedge C; \neg X \wedge \neg Y \wedge \neg Z \wedge \neg C; \neg X \wedge Y \wedge \neg Z \wedge \neg C; X \wedge \neg Y \wedge \neg Z \wedge \neg C \mid B \}$ (based on <D1*>)

Inus-complexes are the central element of a theory of causality based on the presence or absence of circumstances, as inus-complexes combine the notions of causal factors and alternative causes. In the above case X and Y are causal factors of a cause which is alternative to Z (see theorems <T1> and <T2> below).

A.2.6 Homogeneity

Definition: *Context B guarantees homogeneity, iff only conditions that are causally irrelevant to the examined consequent C (and ¬C) can change, (i) except for the examined antecedent A and (ii) conditions X that are causally relevant to C in virtue of A being causally relevant to C.*

A condition X is causally relevant to C in virtue of A being causally relevant to C with respect to a context B, iff in all contexts within B, in which X is causally relevant to C, A is causally relevant to C as well (but not necessarily vice versa).

A.2.6.1 Causal relevance

$A \mathcal{R} C \mid B$

$\Leftrightarrow \exists \{A \wedge C ; \neg A \wedge \neg C \mid B\}$ (based on <D1>)

$\Leftrightarrow A \wedge B \rightarrow C$ and $\neg A \wedge B \rightarrow \neg C$ <D5>

The last equivalence <D5> follows from the definition of homogeneity.

The importance of equivalence <D5> cannot be understated as it guarantees that circumstances that are causally relevant can be used to manipulate a phenomenon.

In other words, iff A is causally relevant to C with respect to background B, then whenever A is present in background B, then C is present and whenever A is absent in background B, then C is absent as well.

A.2.6.2 Causal irrelevance

$A \mathcal{I} C \mid B$

$\Leftrightarrow \exists \{A \wedge C ; \neg A \wedge C \mid B\}$ (based on <D2>)

$\Leftrightarrow A \wedge B \rightarrow C$ and $\neg A \wedge B \rightarrow C$ <D6>

The last equality <D6> follows from the definition of homogeneity.

Thus, changes in causally irrelevant circumstances will never have an impact on the phenomenon.

In other words, iff A is causally irrelevant to C with respect to background B, then whenever A is present in background B, then C is present and whenever A is absent in background B, then C is nevertheless present.

A.2.7 Relevance in conjunction

$A_1 \Diamond A_2 \Diamond \ldots \Diamond A_n \mathcal{R} C \mid B \Leftrightarrow \exists \{A_1 \wedge \ldots \wedge A_n \wedge C; \neg A_1 \wedge \ldots \wedge \neg A_n \wedge \neg C \mid B\}$ <D7>

3 Simple Theorems

The distinction between lemmata and theorems in the following is somewhat arbitrary, but shall convey some general sense of importance.

A.3.1 Lemma 1: $A \mathcal{R} C \mid B \Leftrightarrow \neg A \mathcal{R} \neg C \mid B$ <L1>

Proof: $A \mathcal{R} C \mid B \Leftrightarrow \exists \{A \wedge C ; \neg A \wedge \neg C \mid B\} \Leftrightarrow \exists \{\neg A \wedge \neg C; A \wedge C \mid B\} \Leftrightarrow \neg A \mathcal{R} \neg C \mid B$

The first equivalence holds due to <D1>, the second equivalence holds because the ordering of the instances is arbitrary, the third equivalence again holds due to <D1>.

A.3.2 Lemma 2: $A \ \mathcal{I}\ C \mid B \ \Leftrightarrow\ \neg A \ \mathcal{I}\ C \mid B$ <L2>

Proof: $A \ \mathcal{I}\ C \mid B \ \Leftrightarrow \exists \{A \wedge C ; \neg A \wedge C \mid B \} \ \Leftrightarrow\ \exists \{\neg A \wedge C; A \wedge C \mid B \} \ \Leftrightarrow\ \neg A \ \mathcal{I}\ C \mid B$

The first equivalence holds due to <D2>, the second equivalence holds because the ordering of the instances is arbitrary, the third equivalence again holds due to <D2>.

A.3.3 Lemma 3: $\exists \{A \wedge C ; \neg A \wedge \neg C \mid B \wedge X\} \Leftrightarrow \exists \{A \wedge C \wedge X; \neg A \wedge \neg C \wedge X \mid B \}$ <L3>

This follows from the manner, in which the background is defined.

A.3.4 Lemma 4: $A \ \mathcal{R}_{cf}\ C \mid B \Rightarrow \exists X \rightarrow X \ \mathcal{R}_{cf}\ C \mid B$ <L4>

$A \ \mathcal{R}_{cf}\ C \mid B$

$\Leftrightarrow \exists X \rightarrow A \ \mathcal{R}\ C \mid B \wedge X$ and $A \ \mathcal{I}\ \neg C \mid B \wedge \neg X$ (based on <D3>)

$\Leftrightarrow \exists X \rightarrow \exists \{A \wedge X \wedge C; \neg A \wedge X \wedge \neg C; A \wedge \neg X \wedge \neg C; \neg A \wedge \neg X \wedge \neg C \mid B \}$
(<D1>, <D2>, <L3>)

$\Leftrightarrow \exists X \rightarrow X \ \mathcal{R}\ C \mid B \wedge A$ and $X \ \mathcal{I}\ \neg C \mid B \wedge \neg A$ (<L3>, <D1>, <D2>)

$\Rightarrow \exists X \rightarrow X \ \mathcal{R}_{cf}\ C \mid B$ (<D3>)

Thus, as is plausible, if A is relevant as a causal factor to C, then there exists a complementary causal factor X, wherein $A \wedge X \ \mathcal{R}\ C \mid B$ (see theorem <T1>).

A.3.5 Lemma 5: $A \ \mathcal{R}_{ac}\ C \mid B \Rightarrow \exists X \rightarrow X \ \mathcal{R}_{ac}\ C \mid B$ <L5>

$A \ \mathcal{R}_{ac}\ C \mid B$

$\Leftrightarrow \exists X \rightarrow A \ \mathcal{R}\ C \mid B \wedge \neg X$ and $A \ \mathcal{I}\ C \mid B \wedge X$ (based on <D4>)

$\Leftrightarrow \exists X \rightarrow \exists \{A \wedge \neg X \wedge C; \neg A \wedge \neg X \wedge \neg C; A \wedge X \wedge C; \neg A \wedge X \wedge C \mid B \}$
(<D1>, <D2>, <L3>)

$\Leftrightarrow \exists X \rightarrow X \ \mathcal{R}\ C \mid B \wedge \neg A$ and $X \ \mathcal{I}\ C \mid B \wedge A$ (<L3>, <D1>, <D2>)

$\Rightarrow \exists X \rightarrow X \ \mathcal{R}_{ac}\ C \mid B$ (<D3>)

Thus, as is plausible, if A is relevant as an alternative cause to C, then there exists a complementary alternative cause X, wherein A ∨ X $\mathcal{R}$ C | B (see theorem <T2>).

A.3.6 Theorem 1: $A\ \mathcal{R}_{cf}\ C \mid B \Leftrightarrow \exists\ X \rightarrow A \wedge X\ \mathcal{R}\ C \mid B$ <T1>

This theorem together with theorem 2 is important as it shows that the notion of causal relevance is scalable, i.e. can be employed at different levels of ∧ and ∨ combinations, if the background is adjusted accordingly.

Proof: $A\ \mathcal{R}_{cf}\ C \mid B$

⇔ ∃ X → A $\mathcal{R}$ C | B ∧ X and A $\mathcal{I}$ ¬C | B∧¬*X* (based on <D3>)

⇔ ∃ X → ∃ {A ∧ C ; ¬A ∧ ¬C | B ∧ X} and ∃ {A ∧ ¬C ; ¬A ∧ ¬C | B ∧¬*X*} (<D1>, <D2>)

⇔ ∃ X → ∃ {A ∧ C ∧ X; ¬A ∧ ¬C ∧ X | B} and ∃ {A ∧ ¬C ∧ ¬X; ¬A ∧ ¬C ∧ ¬X | B} (<L3>)

⇔ ∃ X → ∃ {A ∧ C ∧ X; ¬A ∧ ¬C ∧ X; A ∧ ¬C ∧ ¬X; ¬A ∧ ¬C ∧ ¬X | B}

⇔ ∃ X → ∃ {A ∧ X ∧ C; ¬A ∧ X ∧ ¬C; A ∧ ¬X ∧ ¬C; ¬A ∧ ¬X ∧ ¬C | B}

⇔ A ∧ X $\mathcal{R}$ C | B (<D1*>)

q.e.d.

A.3.7 Theorem 2: $A\ \mathcal{R}_{ac}\ C \mid B \Leftrightarrow \exists\ X \rightarrow A \vee X\ \mathcal{R}\ C \mid B$ <T2>

This theorem together with theorem 1 is important as it shows that the notion of causal relevance is scalable, i.e. can be employed at different levels of ∧ and ∨ combinations, if the background is adjusted accordingly.

Proof: $A\ \mathcal{R}_{ac}\ C \mid B$

⇔ *A* $\mathcal{R}$ *C* | *B*∧¬*X* and *A* $\mathcal{I}$ *C* | *B*∧*X* (based on <D4>)

⇔ ∃ X → ∃ {A ∧ C ; ¬A ∧ ¬C | B ∧ ¬X} and ∃ {A ∧ C ; ¬A ∧ C | B ∧ *X*} (<D1>, <D2>)

⇔ ∃ X → ∃ {A ∧ C ∧ ¬X; ¬A ∧ ¬C ∧ ¬X | B} and ∃ {A ∧ C ∧ X; ¬A ∧ C ∧ X | B} (<L3>)

⇔ ∃ X → ∃ {A ∧ C ∧ ¬X; ¬A ∧ ¬C ∧ ¬X; A ∧ C ∧ X; ¬A ∧ C ∧ X | B}

⇔ ∃ X → ∃ {A ∧ X ∧ C; ¬A ∧ X ∧ C; A ∧ ¬X ∧ C; ¬A ∧ ¬X ∧ ¬C | B}

⇔ A ∨ X $\mathcal{R}$ C | B (<D1**>)

q.e.d.

A.3.8 Theorem 3:

$A\ \mathcal{I}\ C \mid B \wedge X$ *and* $A\ \mathcal{I}\ C \mid B \wedge \neg X \rightarrow A\ \mathcal{I}\ C \mid B \wedge x$, *if* $X, \neg X$ *are all possible values of* x.[2]

$A\ \mathcal{I}\ C \mid B \wedge X$ *and* $A\ \mathcal{I}\ \neg C \mid B \wedge \neg X \rightarrow a\ \mathcal{I}^*\ c \mid B \wedge x$, *if* $A, \neg A$ *are all possible values of* a; $C, \neg C$ *all possible values of* c; *and* $X, \neg X$ *all possible values of* x.

$A\ \mathfrak{R}\ C \mid B \wedge X$ *and* $A\ \mathfrak{R}\ C \mid B \wedge \neg X \rightarrow A\ \mathfrak{R}\ C \mid B \wedge x$, *if* $X, \neg X$ *are all possible values of* x.

These theorems are important, because they illustrate how relevance and irrelevance statements become more general, i.e. how the background can be extended, when further evidence is collected.

Proof: The first part is obvious.

For the second part, note that a specific notation irrelevance* needed to be introduced in the second case because a circumstance X relevant to C changed in the context.

$A\ \mathcal{I}\ C \mid B \wedge X$ *and* $A\ \mathcal{I}\ \neg C \mid B \wedge \neg X$

$\Leftrightarrow A\ \mathcal{I}\ C \mid B \wedge X,\ \neg A\ \mathcal{I}\ C \mid B \wedge X,\ A\ \mathcal{I}\ \neg C \mid B \wedge \neg X$ *and* $\neg A\ \mathcal{I}\ \neg C \mid B \wedge \neg X$

Iff these four irrelevance statements are true, then by definition $a\ \mathcal{I}^*\ c \mid B \wedge x$ is true, where $X\ \mathfrak{R}\ C \mid B \wedge a$.

Regarding the third part:

$A\ \mathfrak{R}\ C \mid B \wedge X$ *and* $A\ \mathfrak{R}\ C \mid B \wedge \neg X$

$\Leftrightarrow \exists \{A \wedge X \wedge C; \neg A \wedge X \wedge \neg C; A \wedge \neg X \wedge C; \neg A \wedge \neg X \wedge \neg C \mid B\}$ (based on <D1> and <L3>)

$\Leftrightarrow X\ \mathcal{I}\ C \mid B \wedge A$ *and* $X\ \mathcal{I}\ \neg C \mid B \wedge \neg A$

$\Leftrightarrow x\ \mathcal{I}^*\ c \mid B \wedge a$ (based on second part above)

$\Rightarrow A\ \mathfrak{R}\ C \mid B \wedge x$

The last step follows, because x is irrelevant* to c with respect to background $B \wedge a$.

A.3.9 Theorem 4: $A \wedge X\ \mathfrak{R}\ C \mid B \Rightarrow A\ \mathfrak{R}\ C \mid B \wedge X$; $A \wedge X\ \mathfrak{R}\ C \mid B \Rightarrow A\ \mathcal{I}\ \neg C \mid B \wedge \neg X$; $A \vee X\ \mathfrak{R}\ C \mid B \Rightarrow A\ \mathcal{I}\ C \mid B \wedge X$; $A \vee X\ \mathfrak{R}\ C \mid B \Rightarrow A\ \mathfrak{R}\ C \mid B \wedge \neg X$

These relationships follow directly from the definitions <D1*> and <D1**>. They illustrate, how relationships for a narrower background can be derived from relationships for a wider background.

[2] There is no double arrow, since it is generally not required that all possible combinations in the background must actually occur.

A.3.10 Theorem 5: $A_1 \lozenge A_2 \lozenge \ldots \lozenge A_n\ \mathcal{R}\ C \mid B \Leftrightarrow \exists \{A_1 \wedge \ldots \wedge A_n \wedge C; \neg A_1 \wedge \ldots \wedge \neg A_n \wedge \neg C \mid B\}$ $\Rightarrow$ there is an inus-complex among the A, which is causally relevant to C with respect to B.

The first equivalence is definition <D7>. Otherwise, the theorem follows from the assumptions of homogeneity and determinism.

The theorem illustrates an important intuition that, while at first irrelevant circumstances are co-varied with the actual causes, one can narrow down to the actual causes by collecting further variational evidence. The theorem ensures that an expression that is causally relevant exists among the covarying circumstances. For example, one might first observe an instance where a virus and full moon are present, when a disease is present, and a further instance where the virus and full moon are absent, when the disease is absent. Further inquiry and collection of instances will then determine that the virus and not the full moon causes the disease.

A.3.11 Transitivity: $X\ \mathcal{R}\ Y \mid B$ and $Y\ \mathcal{R}\ Z \mid B \rightarrow X\ \mathcal{R}\ Z \mid B$

Proof: $X\ \mathcal{R}\ Y \mid B$ and $Y\ \mathcal{R}\ Z \mid B$

$\Leftrightarrow$ $X \wedge B \rightarrow Y$, $\neg X \wedge B \rightarrow \neg Y$, $Y \wedge B \rightarrow Z$ and $\neg Y \wedge B \rightarrow \neg Z$ (based on <D5> and <D6>)

$\Rightarrow$ $X \wedge B \rightarrow Y \wedge B \rightarrow Z$, $\neg X \wedge B \rightarrow \neg Y \wedge B \rightarrow \neg Z$ (B can be added to Y and ¬Y, because the background will be present as well)

$\Rightarrow$ $X \wedge B \rightarrow Z$, $\neg X \wedge B \rightarrow \neg Z$

$\Leftrightarrow$ $X\ \mathcal{R}\ Z \mid B$

Under certain conditions, the notions of causal factor, alternative cause and inus-complex are also transitive (see Section 5.3.3 of Pietsch 2022). Transitivity, like the possibility to look at causal relationships at different levels of coarse-graining, are crucial tools for keeping track of complex causal relationships.

4 Notation

small Latin letters a, c, x	variables
capital Latin letters A, C, X (except B)	states
A, C, X	shorthand for a = A, c = C, x = X
¬A, ¬C, ¬X	shorthand for a = ¬A, c = ¬C, x = ¬X
a, a_i, A, A_i	antecedent (circumstance)

c, C_i	consequent (phenomenon)
x, X, y, Y	further variables
∧, e.g. A ∧ X	And, e.g. both states A and X are realized
∨, e.g. A ∨ X	Or, e.g. states A or X are realized (or both)
B	background or context (terms used synonymously)
B ∧ X	background including B and x = X
B ∧ x	background including B and where x can take on any value, e.g. X or ¬X in case of a binary variable
$A\ \mathcal{R}\ C \mid B$	A is causally relevant to C with respect to background B
$A\ \mathcal{R}_{cf}\ C \mid B$	A is relevant as a causal factor to C with respect to background B
$A\ \mathcal{R}_{ac}\ C \mid B$	A is relevant as an alternative cause to C with respect to background B
$A\ \mathcal{I}\ C \mid B$	A is causally irrelevant to C with respect to background B
∃ {A ∧ C ; ¬A ∧ ¬C | *B* }	there exists an instance A ∧ C with background B, a further instance ¬A ∧ ¬C also with background B and B guarantees homogeneity
∃ X →	there exists an X so that
A ∧ B → C	if A ∧ B, then C

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